



Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2024

Further Mathematics

Assessment Unit AS 2

assessing

Applied Mathematics

MV18

[SFM21]

FRIDAY 17 MAY, AFTERNOON

Time

1 hour 30 minutes, plus your additional time allowance.

Instructions to Candidates

Write your Centre Number and Candidate Number on the **Answer Booklet** provided.

You must answer the questions in the dedicated spaces provided in the Answer Booklet.

Do not write on blank pages.

Complete in black ink only.

Questions which require drawing or sketching should be completed using an HB pencil.

You must answer **all** questions from sections A and B **or** A and C **or** A and D **or** C and D.

You should spend equal time on each of the two sections.

Show clearly the full development of your answers. **Answers without working may not gain full credit.**

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

Information for Candidates

The total mark for this paper is 100.

The total mark for each section of this paper is 50.

Figures in brackets printed at the end of each question indicate the marks awarded to each question or part question.

Answers should include diagrams where appropriate and marks may be awarded for them.

Take $g = 9.8 \text{ m s}^{-2}$, unless specified otherwise.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$.

Section A Mechanics 1

Answer all five questions in this section.

- 1** Mojo the monkey, of mass 5 kg, drops vertically from a point A on a branch whilst holding one end of a stretchable vine.

The other end of the vine is attached to the point A.

Model Mojo as a particle initially at rest.

Model the vine as an elastic string of natural length 8 m with modulus of elasticity 128 N.

Find how far the vine extends before Mojo first comes to instantaneous rest. [8 marks]

- 2** A truck of mass 2200 kg, with its engine switched off, can freewheel at a constant speed of 25 m s^{-1} down a slope.

The slope is inclined at 12° to the horizontal.

- (i)** Find the resistance to its motion. [2 marks]

The resistance is proportional to the square of the speed.

- (ii)** Find the constant of proportionality. [2 marks]

The truck turns around and accelerates up the same slope at 0.8 m s^{-2}

You may assume the same relationship between resistance and speed.

- (iii)** Find the power output of its engine at the instant when it is travelling at 15 m s^{-1} [7 marks]

3 (a) A force

$$\mathbf{F} = (5\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})\text{N}$$

acts on a particle.

The particle moves from a point A with position vector

$$\overrightarrow{\text{OA}} = -4\mathbf{i} + 6\mathbf{j} + \alpha\mathbf{k}$$

to a point B with position vector

$$\overrightarrow{\text{OB}} = -\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$$

Find the value of α if the force \mathbf{F} does no work.
[4 marks]

(b) A particle P is moving along a horizontal line through O.

The distance from O to P is x metres.

A variable force, G newtons, acts on P, where

$$G = \frac{20}{3} \left(3x - \frac{16}{x^3} \right)$$

(i) Find the work done by G when the particle P moves from $x = 1$ to $x = 4$ [5 marks]

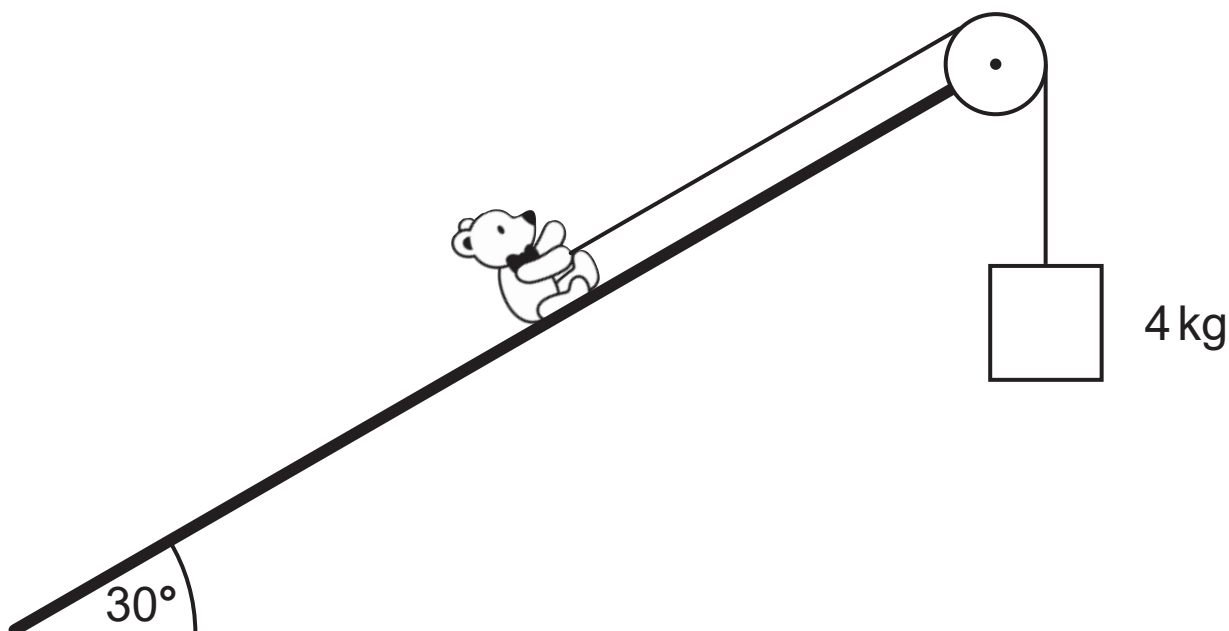
The particle P has mass 0.5 kg.

When $x = 1$ it is moving with a speed of 21 m s^{-1}

(ii) Find its speed when $x = 4$ [2 marks]

- 4 A wooden teddy bear is connected to a block by a light, inextensible string passing over a smooth fixed pulley as shown in **Fig. 1** below.

Fig. 1



The bear has mass 1.5 kg and rests on a rough plane inclined at 30° to the horizontal.

The block has mass 4 kg .

Model both the bear and the block as particles.

The bear and the block are initially at the same horizontal level.

This level should be taken as the baseline for gravitational potential energy.

Both bodies are released from rest and move without obstruction.

The block drops 0.4 m , reaching a speed of $v\text{ m s}^{-1}$

- (i) Find the change in the total gravitational potential energy of both particles. [4 marks]
- (ii) Find, in terms of v , the change in total kinetic energy of both particles. [2 marks]

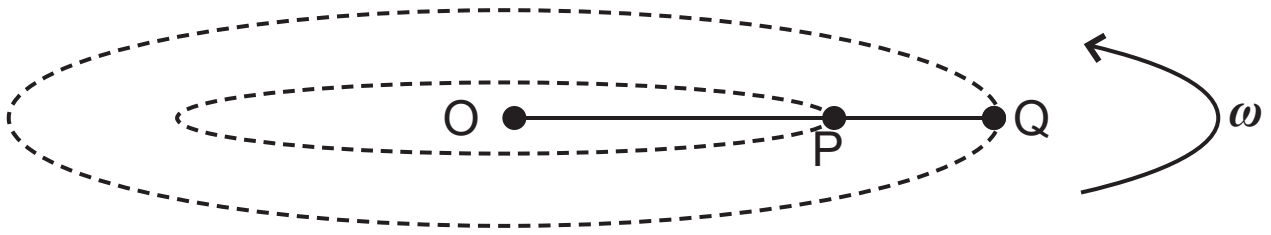
The work done by friction on the bear is -1.70 J .

- (iii) Using the Work-Energy Principle, find the value of v . [3 marks]

- 5 **Fig. 2** below shows particles P and Q, of masses 0.4 kg and 0.6 kg respectively, attached by a light inextensible string to a fixed point O.

P and Q are moving in horizontal circles, centre O, such that O, P and Q lie on a straight line.

Fig. 2



P and Q are moving on a horizontal, frictionless surface with constant angular velocity ω .

The tension in the string OP is 2.16 N and the tension in the string PQ is 1.68 N.

The lengths of OP and OQ are r_1 and r_2 metres respectively.

(i) Show $\omega^2 r_2 = 2.8 \text{ m s}^{-2}$ [3 marks]

(ii) Show $\omega^2 r_1 = 1.2 \text{ m s}^{-2}$ [4 marks]

(iii) Given that the velocity of P is 1.5 m s^{-1} , find the velocity of Q. [4 marks]

Section B Mechanics 2

Answer all five questions in this section.

- 1 [In this question take \mathbf{i} and \mathbf{j} to be unit vectors due East and due North respectively.]

A tractor is moving in a straight line with velocity $(3.7\mathbf{i} - 4.6\mathbf{j})\text{ms}^{-1}$

A rabbit sits at the origin O.

The rabbit's burrow has position vector $(24\mathbf{i} + 7\mathbf{j})\text{m}$.

At the instant when the tractor is 16 m due North of the rabbit, the rabbit starts to run in a straight line towards its burrow at a constant speed of 5ms^{-1}

Model the tractor and rabbit as particles.

- (i) Find the velocity of the rabbit. [2 marks]
- (ii) Find the velocity of the tractor relative to the rabbit. [2 marks]
- (iii) Find how close the tractor will come to the rabbit. [6 marks]

- 2** A satellite moves in a circular orbit 600 km above the Earth's surface.

The radius of the Earth is 6.37×10^6 m.

Find the speed of the satellite. [6 marks]

- 3 The gaps between crests of waves are described by the equation

$$xk = \omega t$$

where x is a distance, t is a time and ω is a frequency with dimensions T^{-1}

- (i) Using the equation above, find the dimensions of the wave number, k . [2 marks]
- (ii) Show that the equation for gravity waves in deep water

$$\omega^2 = gk$$

is dimensionally consistent, where g is the acceleration due to gravity. [2 marks]

The general dispersion relation for waves in deep liquid states

$$\omega^2 \propto \rho^a g^b k^c \gamma^d$$

where ρ is the density of the liquid with dimensions ML^{-3}

k is the wave number, and

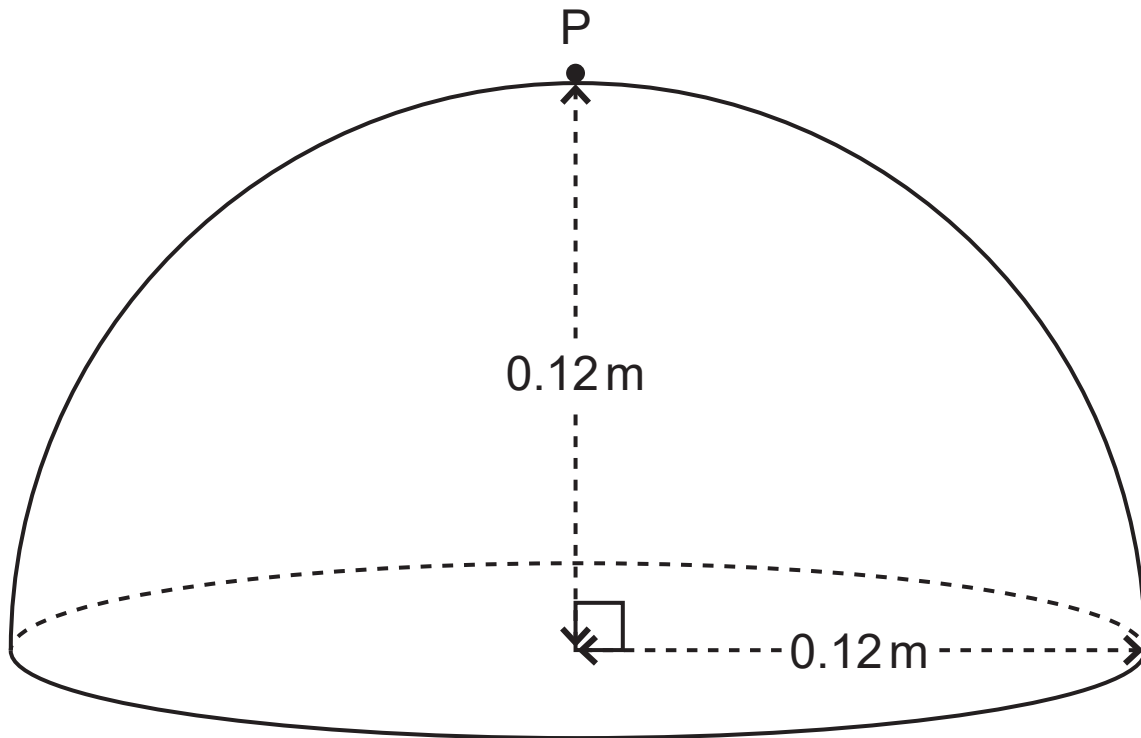
γ is the surface tension of the liquid, with dimensions MT^{-2}

(iii) Use the Method of Dimensions to determine a , b and c in terms of d . [7 marks]

(iv) Hence identify a dimensionless combination of the 4 quantities. [2 marks]

- 4 **Fig. 1** below shows a smooth hemisphere of radius 0.12 m with a horizontal base.

Fig. 1



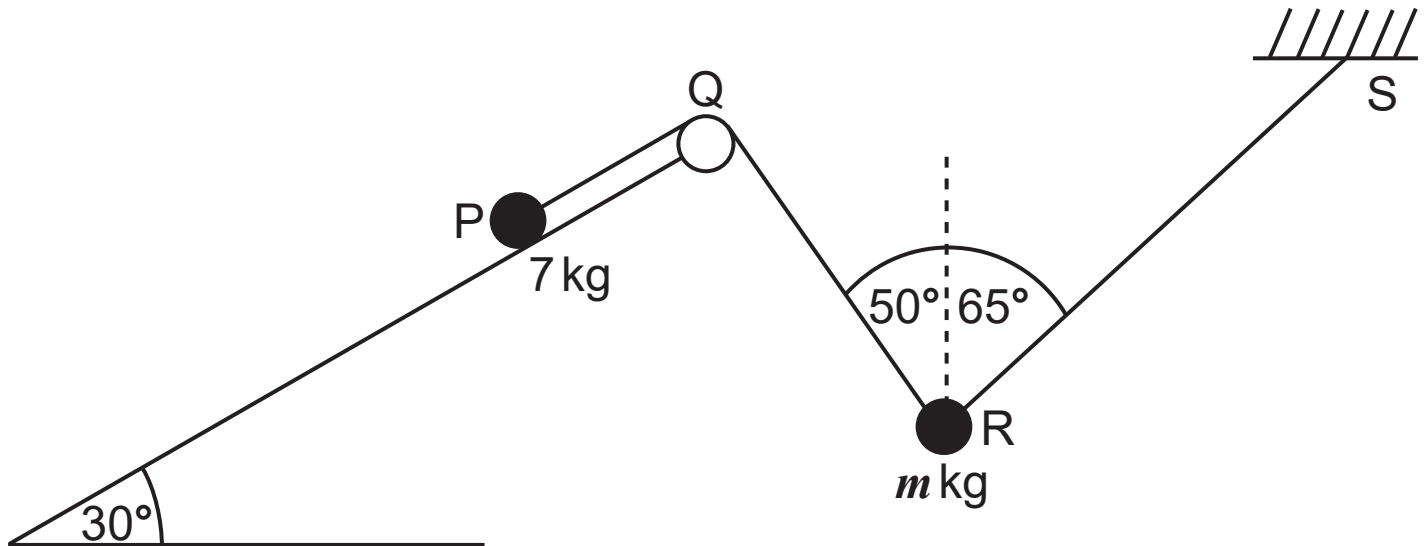
A smooth particle P sits at rest at the top of the hemisphere.

P is then displaced slightly and slips down the surface of the hemisphere.

Find the distance travelled by the particle before it leaves the surface. [9 marks]

- 5 Fig. 2 below shows a block P, of mass 7 kg, at rest on a smooth plane which is inclined at 30° to the horizontal.

Fig. 2



P is attached by a light inextensible string, passing over a smooth fixed pulley Q, to a block R of mass m kg.

R is attached to a fixed point S by a taut elastic string of natural length 2.8 m and modulus of elasticity 98 N.

Model P and R as particles.

P, Q, R and S all lie in the same vertical plane.

RQ and RS are inclined at 50° and 65° to the vertical respectively.

The system is in equilibrium.

- (i) Draw a diagram showing all the forces acting on each of the blocks. [2 marks]
- (ii) Find the extension in the elastic string RS. [7 marks]
- (iii) Find the mass of R. [3 marks]

Section C Statistics

Answer all five questions in this section.

- 1** An educationalist would like to study the effects of screen time on children of nursery school age.

The educationalist intends to visit her local nursery school to hold interviews with the pupils and parents of a particular class.

- (i)** State the sampling technique that is being used for this investigation. [1 mark]

- (ii)** Give one advantage and one disadvantage of using this particular sampling technique. [2 marks]

- (iii)** State why this sampling technique is referred to as non-random. [1 mark]

- 2 The discrete random variable X has the probability distribution given in **Table 1** below.

Table 1

| | | | | | |
|------------|--------|-------|-------|-------|-------|
| x | -3.5 | b | c | 3.5 | 7 |
| $P(X = x)$ | a | 0.2 | 0.1 | 0.2 | 0.1 |

The expected value of X is 0

The expected value of X^2 is 12.85

(i) Find the value of a . [1 mark]

(ii) Given that b is a negative integer, find the values of b and c . [6 marks]

(iii) Find the variance of X . [1 mark]

The random variable Y is related to X by the formula
 $Y = 0.2X - 2.1$

(iv) Find the mean and standard deviation of Y . [3 marks]

- 3 A scientist is investigating the link between ambient air pressure, y kPa, and the height above sea level, x km, in the lowest layer of the Earth's atmosphere.

The data is given in **Table 2** below.

Table 2

| | | | | | |
|---------------------------|----|----|----|-----|----|
| Height (x , km) | 1 | 2 | 3 | 4 | 5 |
| Air pressure (y , kPa) | 88 | 83 | 72 | q | 49 |

Using this data the scientist calculated the regression coefficient of y on x to be $b = -10.3$

The air pressure at a height of 4 km was used in this calculation of the regression coefficient. However, the value was then inadvertently deleted.

- (i) Find the value of q that was used in the original calculation. [7 marks]
- (ii) Give a physical interpretation of the parameter b in the context of this question. [2 marks]
- (iii) Use the equation of the least squares regression line to find the air pressure at a height of 2.4 km. [4 marks]
- (iv) Comment on the reliability of this result. [1 mark]

- 4 The latest speech recognition software incorrectly identifies, on average, 4.5 words per page of text.
- (i) Give two assumptions that would need to be made for the Poisson distribution to be a suitable model in this context. [2 marks]
- (ii) Find the probability that, in one page of text, there are at least 8 incorrectly identified words. [3 marks]

The developer is continually improving the software.

Her aim, in a randomly selected page of text, is that the probability of more than 3 words being incorrectly identified is less than 0.2

- (iii) Using the Poisson Cumulative Distribution Function tables, find the largest average number of incorrectly identified words (per page of text) that achieves this aim. Give your answer to the nearest whole number. [4 marks]

- 5 (a) The World Space Agency Astronaut Training programme consists of 6 astronauts from USA, 5 astronauts from China and 3 astronauts from Europe.

A team of 6 astronauts is to be randomly selected, from this pool of 14 astronauts, to take part in humanity's first manned mission to Mars.

Find the number of different ways of selecting the team of 6 astronauts for the mission if:

(i) there are no restrictions; [2 marks]

(ii) the team must consist of exactly 4 astronauts from USA. [3 marks]

(b) Each letter of the word **statistic** is written on a separate card.

The word **statistic** contains 3 vowels (**a** and **i**) and 6 consonants.

The 9 cards are to be arranged in a random order in a line.

(i) Find the number of different arrangements of letters possible if there are no restrictions. [3 marks]

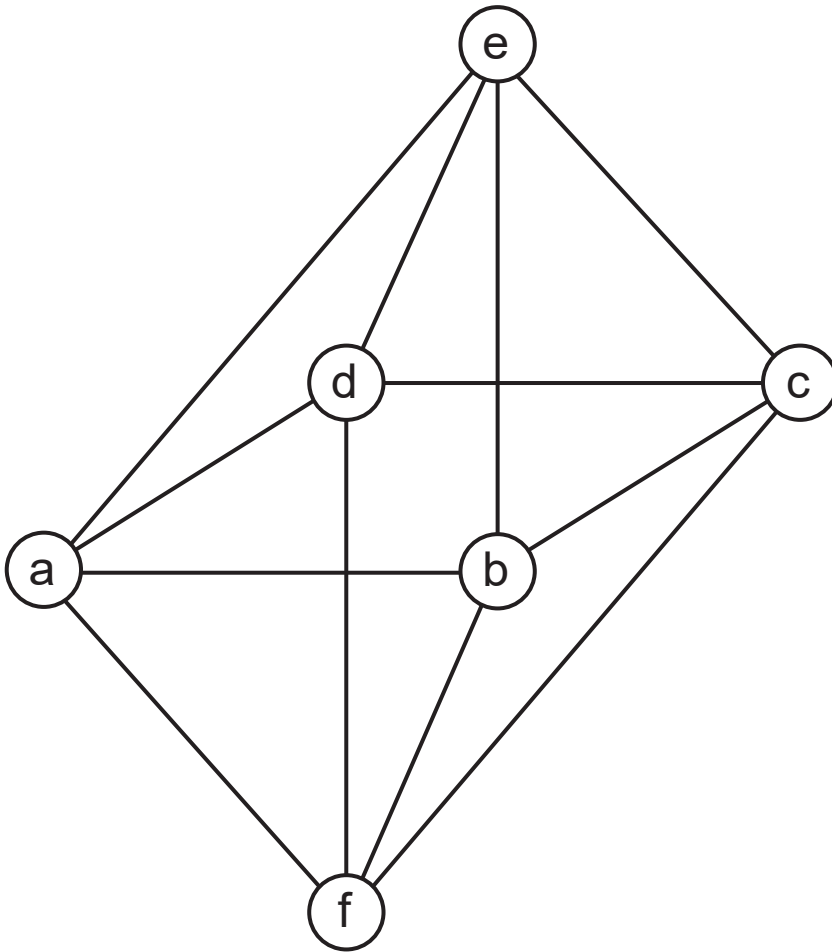
(ii) Find the probability that the 9 cards appear with the 3 vowels together in a group. [4 marks]

Section D Discrete and Decision Mathematics

Answer all five questions in this section.

1 Fig. 1 below shows the graph G with labelled vertices.

Fig. 1



(i) List a Eulerian circuit of G . [2 marks]

K_n is the complete graph on n vertices.

(ii) Which of K_3 , K_4 or K_5 are subgraphs of G ? [3 marks]

(iii) List the edges that must be added to G to form K_6
[2 marks]

- 2 (i)** Show that $(x - 1)$ is a factor of $x^3 - 6x^2 + 11x - 6$
[2 marks]

u_n satisfies the recurrence relationship

$$u_{n+3} = 6u_{n+2} - 11u_{n+1} + 6u_n$$

where

$$n \geq 0 \quad \text{and} \quad u_0 = 6, \quad u_1 = 7 \quad \text{and} \quad u_2 = 11$$

- (ii)** Show that

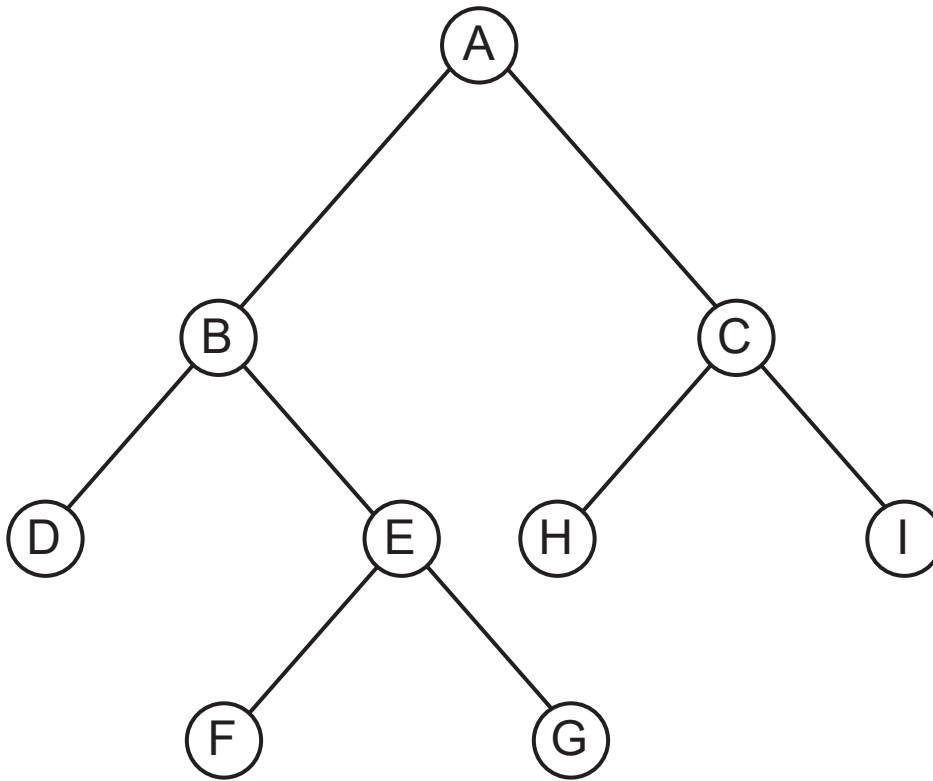
$$u_n = P\alpha^n + Q\beta^n + R$$

where P , Q , R , α and β are integers to be determined.
[7 marks]

- (iii)** Hence find u_{14} [1 mark]

3 (a) Fig. 2 below shows a binary tree with root node A.

Fig. 2



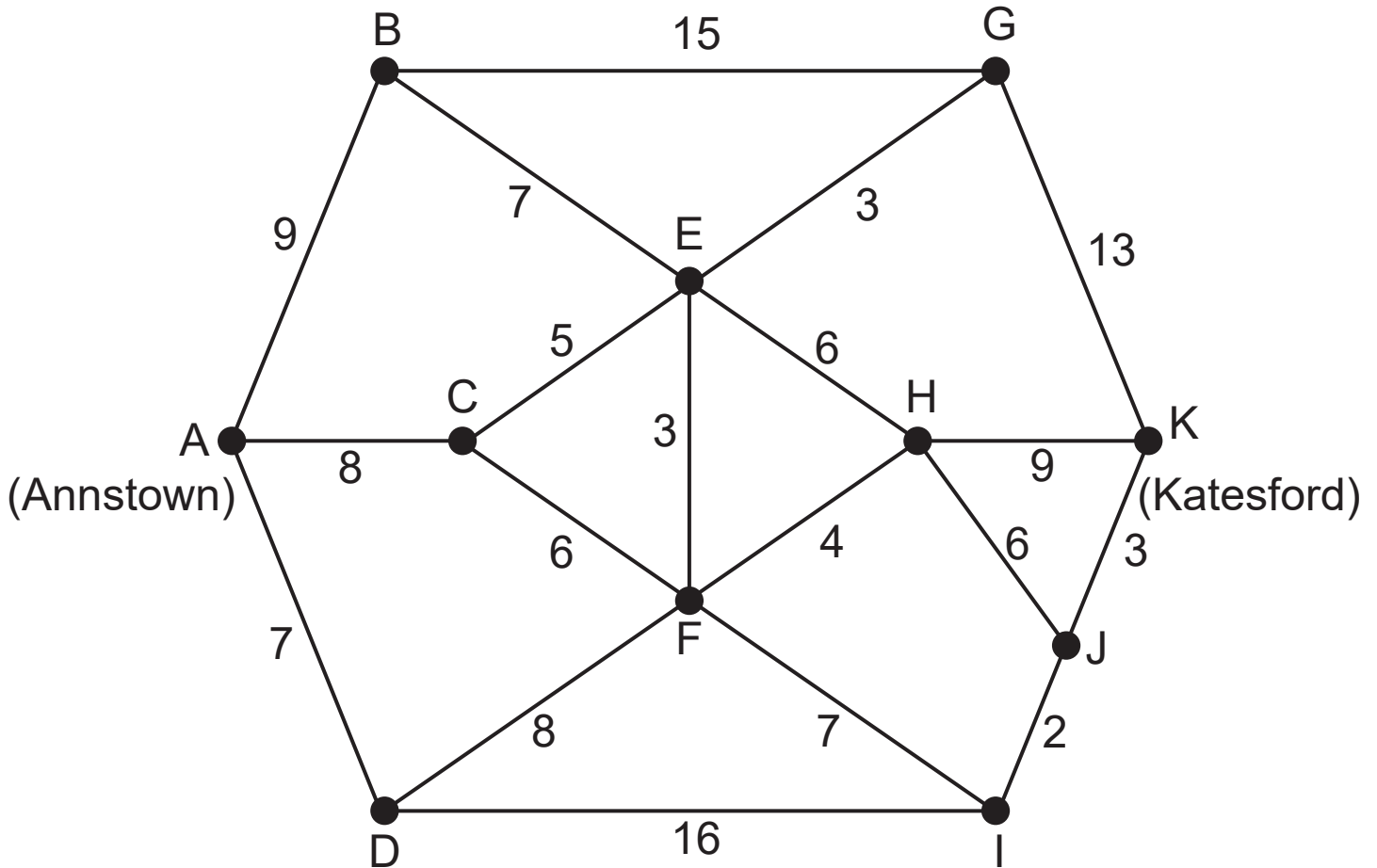
List the order in which the nodes are visited in:

(i) a breadth first search; [1 mark]

(ii) a depth first search. [1 mark]

(b) The travel times in minutes between 11 towns are given in the road network in **Fig. 3** below.

Fig. 3



A journey is planned from Annstown (vertex A) to Katesford (vertex K).

- (i) Apply Dijkstra's algorithm to this network to calculate the shortest journey time from Annstown to Katesford.

Complete **Diagram 1** in the **Answer Booklet**.
[5 marks]

(ii) List the route with the shortest journey time.
[2 marks]

(iii) State the shortest journey time. [1 mark]

4 Let p , q and r be propositional statements.

The logical operation p nand q is defined as

$$p \text{ nand } q = \sim (p \text{ and } q)$$

with the truth table shown in **Fig. 4** below.

Fig. 4

| p | q | p and q | p nand q |
|-----|-----|-------------|--------------|
| T | T | T | F |
| T | F | F | T |
| F | T | F | T |
| F | F | F | T |

(i) Write out the truth table for p nand p . [2 marks]

(ii) Use truth tables to prove that the compound statements

$$(p \text{ nand } q) \text{ nand } (p \text{ nand } r)$$

and

$$p \text{ and } (q \text{ or } r)$$

are equivalent. [8 marks]

- 5 The group table for the rotation symmetry group of the regular octagon C_8 is given in **Fig. 5** below.

Fig. 5

| \circ | i | r | r^2 | r^3 | r^4 | r^5 | r^6 | r^7 |
|---------|-------|-------|-------|-------|-------|-------|-------|-------|
| i | i | r | r^2 | r^3 | r^4 | r^5 | r^6 | r^7 |
| r | r | r^2 | r^3 | r^4 | r^5 | r^6 | r^7 | i |
| r^2 | r^2 | r^3 | r^4 | r^5 | r^6 | r^7 | i | r |
| r^3 | r^3 | r^4 | r^5 | r^6 | r^7 | i | r | r^2 |
| r^4 | r^4 | r^5 | r^6 | r^7 | i | r | r^2 | r^3 |
| r^5 | r^5 | r^6 | r^7 | i | r | r^2 | r^3 | r^4 |
| r^6 | r^6 | r^7 | i | r | r^2 | r^3 | r^4 | r^5 |
| r^7 | r^7 | i | r | r^2 | r^3 | r^4 | r^5 | r^6 |

The operator \circ denotes the composition of rotations.

r is a positive rotation of 45° and i is the identity.

(i) List the elements of C_8 and their periods. [2 marks]

(ii) List all the subgroups of C_8 [3 marks]

C_{12} is the rotational symmetry group of the regular dodecagon. Its generator is ω , a positive rotation of 30°

(iii) Write down a four-element subgroup of C_8 and a four-element subgroup of C_{12} with a mapping between them defining an isomorphism.

Note that the group table for C_{12} is not required.
[3 marks]

The set $G = \{1, 7, 9, 15\}$ forms a group (G, \times_{16}) under multiplication modulo 16

(iv) Complete in the **Answer Booklet** the group table started in **Fig. 6** below. [2 marks]

Fig. 6

| \times_{16} | 1 | 7 | 9 | 15 |
|---------------|---|---|----|----|
| 1 | 1 | 7 | 9 | 15 |
| 7 | | | 15 | |
| 9 | | | | |
| 15 | | | | |

(v) State, with reasons, whether (G, \times_{16}) is isomorphic to the four-element subgroups of C_8 and C_{12} used in part (iii) above. [3 marks]

This is the end of the question paper

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